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WHAT WE THINK WE KNOW ABOUT NEWTON'S *PRINCIPIA*

O QUE NÓS PENSAMOS SOBRE O *PRINCIPIA* DE NEWTON

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ABSTRACT: In this article we examine some of the most controversial points in Newton's *Principia* regarding the contemporary interpretation: the assumed circularity of Definition I, the difference between *vis insita* and *vis inertia*, Law I and Law II. The main goal of this work is to offer a wider view of Newton's writings and provide a historic philosophical approach to the physics teachers which will certainly be useful in physics teaching. We also discuss the modern terminology compared to the one used by Newton himself. Our main concern is to aid the deconstruction of preconceived ideas regarding the *Principia* which are fairly common among physics teachers.
Keywords: Physics Teaching. Isaac Newton. Definitions. Laws. Natural Philosophy.

RESUMO: Neste artigo examinamos alguns dos pontos mais controversos dos *Principia* de Newton tendo em vista as suas interpretações contemporâneas: a circularidade assumida na Definição I, a diferença entre *vis insita* e *vis inertia*, a Lei I e a Lei II. O objetivo do trabalho é oferecer uma visão mais ampla dos escritos de Newton e fornecer uma abordagem filosófica-histórica aos professores a qual certamente será útil no ensino da física. Nós também discutimos a terminologia moderna em comparação com a usada pelo próprio Newton. Nossa principal preocupação é ajudar a desconstrução de ideias preconcebidas sobre os *Principia*, bastante comuns entre professores de física.
Palavras-chave: Ensino de Física. Isaac Newton. Definições. Leis. Filosofia Natural.

RESUMEN: En este artículo examinamos algunos puntos polémicos de los *Principia* de Newton bajo la luz de una interpretación contemporánea, a saber: la circularidad del concepto de masa asumida de la Definición I, la diferencia entre vis insita y vis inertia, las Leyes I y II . El principal objetivo de este trabajo es ofrecer una visión más amplia de los escritos de Newton y proporcionar un enfoque filosófico e histórico para los profesores de física. Discutimos también la terminología moderna en comparación con la usada por el propio Newton. Nuestra principal preocupación es ayudar a la deconstrucción de ideas preconcebidas sobre los *Principia* que son bastante comunes entre los profesores de física.
Palabras lhave: Enseñanza de Física. Isaac Newton. Definiciones. Leyes. Filosofía Natural.

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Introduction

Philosophiae Naturalis Principia Mathematica, to which we will make any reference purely as *Principia*, was written by Isaac Newton and first published in 1687. It consists of a work of three books. It states among other concepts and theories, Newton's Laws of Motion, two of which we will discuss in this article, and Newton's Law of universal gravitation.

The *Principia* is considered by many one of the most important published work in the field of Natural Philosophy. It is the foundation of what we study today as Classical Mechanics. And its historical and philosophical importance is definitely worthy discussing.

In this article we take some of the most controversial points in Newton's writings and open them with a not only physical approach but also a historic philosophical one, in order to clear up some preconceived ideas about the *Principia*. Here, the goal is to provide physics teachers and students or enthusiasts with tools and arguments to comprehend the concepts and terms used beyond the common misinterpretations so widely spread. We go through many relevant names in the history of science to explain how Newton got to his conclusions and to give a perspective of that period.

It is not possible to begin the studies of natural philosophy without understanding some concepts and ideas that were developed long before Newton had his hands on the subject. Ideas from Aristotle, Galileo and Descartes, among so many others fundamental contributors to this field were the basis, whether being then proved right or not, to build and solidify the knowledge we have available today.

In some sections of this article it is brought the possibility of comparison between terms and words used at Newton's time and the ones used today. We also discuss about modern books and their interpretations of the *Principia*.

Many people nowadays, especially students in their first years of school, but not only them, having a first contact with science, get the wrong idea about how the development of the knowledge in this field is given. We strongly believe this happens because of the way science is taught.

It is common to see in our society the belief that a certain theory just popped up in the mind of a genius, the person wrote it down, everyone began to use it and then science was done. It is crucial to deconstruct that idea of an exact, sharp, free from polemic science. That thinking makes the subject seem extremely distant from the students' daily lives, something unachievable and untouchable, therefore not interesting. From another perspective, the important thing is to deconstruct the myth of Newton's genius, but it does not mean deconstructing his genius, *per se*. Newton was a singular figure who worked hard and whose achievements came with a great effort, without magic.

Here, we show how several times, if not always, science was built through disagreement, dispute and different ideas. It is more of a process than just anachievement. And diversity is the key word to the growth. Diversity of people, of ideas and of concepts.

The making of knowledge is slow and it is done brick by brick, with a rupture now and then so the construction gets to be revised, rethought and improved. It's not about throwing the old knowledge away, but remodeling it so it fits the dynamics of a certain time and with new fresh eyes we build and rebuild until what we know today.

What one should keep in mind is that even today's knowledge is not and should never be static or bulletproof, the wall of knowledge remains always under construction. It depends on each and everyone of us to give contributions, to question, to doubt and to give answers to the unlimited field of knowledge.

Newton and the Natural Philosophy

In order to discuss Newton's position regarding the Natural Philosophy and how his work was the trigger to a crucial shift in its course, it is essential to understand the perspective of some of his predecessors, particularly Galileo and Descartes. When Galileo employed the concept of *vis impressa* these can be currently understood as the origins of the principle of inertia (GALILEI, 1997) and Descartes stated his three laws of nature, (SLOWIK, 2014) the theoretical framework, in which explanations for the natural phenomena should be sought, was deeply changed. From this moment, causal relations were required to explain the motion of bodies.

Descartes, based on the mechanical philosophy view, established that every and each causal action should take place through contact. This idea was introduced as an hypothesis, that should be legitimated *a posteriori*.

When Newton came across this explanation, he could not be comfortable with it, since he strongly believed that there was no room for hypothesis in the experimental

Philosophy. And by hypothesis, he means anything that can not be deduced from the phenomenological observation. By contradicting Descartes' arguments, Newton reopened the main problem of mechanics. (SLOWIK, 2014). According to Rupert Hall and Marie Boas Hall (2002), Boyle's corpuscular philosoph" was the great inspiration of Cartesian mechanics. This theory defended the idea that every natural process would consist of the movement of bodies and that any natural change of motion would be caused by the direct action of one body over another, one body "pushing" another. Newton and Descartes share this "Corpuscular Philosophy" and it would have been in it that Newton had been inspired.

Trying to find a solution to the question that Newton himself brought to the table, he went towards a new view of a causal explanation to motion, a force that would act upon distance. However, in order to have this idea accepted, he could not leave the scope of the mechanical philosophy, therefore the only plausible exit would be to enlarge its limits.

It was due to this revolutionary idea that the natural philosophy took a new path. About one century after Newton's *Principia* first edition a division of scientific fields begins, basically changing the order from "why" to "how", i.e., the idea was no longer to explain the cause of a given phenomenum, but to give it a description.

Is Definition I cyclic?

From *Principia*:

> Quantity of matter is a measure of matter that arises from its density and volume jointly. If the density of air is doubled in a space that is also doubled, there is four times as much air, and there is six time as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction, and also for all bodies that are condensed in various ways be any causes whatsoever. For the present, I am not taking into account any medium, if there should be any, freely pervading the interstices between the parts of bodies. Furthermore, I mean this quantity whenever I use the term "body" or "mass" in the following pages. It can always be known from a body's weight, for – by making very accurate experiments with pendulums – I have found it to be proportional to the weight, as will be shown below. (NEWTON, 1999, p. 403-404).

This first definition has been the main target of harsh criticism extremely evident on Ernst Mach's *The science of Mechanics: A critical and Historical Account of Its*

Development. Many people, including physics teachers and students, who have read or heard about Newton's definition of mass usually say that it is a circular definition, that is, a fallacy. However, to fully comprehend the meaning of Definition I, one needs to understand more than just the physical concepts in it, it is primordial to have a philosophical and historical perspective.

The conclusion of the circular character of this definition is commonly seen probably because of today's definition of density, which is considered to be mass per unit volume. So, if the quantity of matter is defined as proportional to the product of density and volume, we can clearly see the circularity. (MACH, 1960). But aren't we missing something?

In the comment following the definition, Newton discusses examples of experiments with compression and expansion of gases in a sealed gear. The quantity of matter of these gases does not change as long as there is no way for the gas to scape, but the gas becomes more or less compact by a factor that is directly proportional to the compression or expansion. So, we must not understand "compression" and "expansion" in terms of volume, what might be today's main use. We should read them thinking about how concentrated or sparse that gas is. In a logical analysis of this comment, one is able to identify how the quantity of matter, as stated by Newton, originates from the product of the space occupied by a gas and how compressed or expanded it is.

The circularity could be an argument if and only if there was no other way to define density other than the quotient of the quantity of matter by the volume. But there is another way to define it, what can be seen through a historic-philosophical research. As there is no further mention of density in *Principia*, we may infer that Newton assumes that the readers would understand what density is. And how would that be possible?

According to the atomismo, a natural philosophy that was developed in many different cultures, nature consists of atoms, that would be an indivisible part of matter, and void. In the West, the atomism began in the 5th century b.C. with Leucippus and his pupil Democritus. As stated in the atomistic view, since there are only atoms and void, every body that is not in a atomic scale, has a fraction full of matter, atoms, while the rest is void. Therefore, density is understood as this fraction, which is constant for a homogeneous body. Hence, the quantity of matter being originated as the product of this fraction and the volume. (DIJKSTERHUIS, 1961). However, this argument does not prevent the circularity of the definition of mass. Simply Dijksterhuis replaces mass density with a fraction of particles which is equivalent to particle density or mass density. The relevant point in the Definition I of the *Principia* is that Newton had dismissed the cartesian ideas, characterized by some kind of invisible means that permeates matter.

Also, in order not to be inclined to have a simpleminded conclusion, one should be aware of Newton's linguistic choice, which differs from the actual wording of the other definitions. All of the other definitions, with the exception of Definition I, begin by stating the term, or concept, that he wants to define followed by the verb "to be". As an example: "The innate force of matter is [...]" "The impressed force is [...]" or "The motive magnitude of the centripetal force is..." The standard form as found in Euclidian style.

Let us take a little while to understand the influence of Euclid's systematization in Newton's work before we begin to analyse Definition I in terms of linguistic. According to the Aristotelian principles of axiomatization, one needs to find clear statements, or at least plausible ones, that can serve as starting-points, these should be preceded by a group of definitions in which the words and terms used require no further explanation. Newton found himself in a position where he should attempt the axiomatization of mechanics, because the employment of notions based on day-to-day experience soon presented itself as inadequate for an exact treatment of the subject. Euclid's Elements seems to have been the model to *Principia*, although Newton didn't achieved Euclid's logical excellence, (DIJKSTERHUIS, 1961) which may be the reason for so many misinterpretations of his first definition.

Now that you might have realized the importance of the word usage in a definition, we are able to discuss the fact that Definition I can be understood as having two parts.

The first one is simply the very beginning of the sentence, which is written in the tradition form expected of a definition, i.e., using the verb "to be": "quantity of matter is a measure". However, in the second part, Newton uses the verb "to arise", as we can see: "that arises from its density and volume jointly". Since the latter is clearly not presented in the primary form in which definitions are usually cast, we may infer that this is, in fact, the statement of a rule; a relation between mass (measure of matter) the new concept, and the, already known, concepts of volume and density. The reader may argue that this is just another assumption, but this is corroborated by the fact that, after so many stages of evolution of the definitions which demonstrates Newton's care in revising and rewriting them, this choice of words prevailed. So it is natural to conclude that the form of Definition I differing from all the others is not just a mere case of accident or style, but rather a deliberate choice. In the end, Newtonian circularity was a deliberate choice.

Vis insita vs Vis inertia

In Definition III Newton writes about a *vis insita*, commonly translated as an "innate force" the *vis inertia*. But to be able to discuss what he might have meant by stating that the difference between this force and inactivity of the "mass" lies in the manner of conceiving it, we shall acknowledge what was previously understood as *vis insita* and *vis inertia*.

Many *Principia* readers find themselves astonished when coming across the word "force" to designate what in modern physics is known solely by inertia. According to Newton, this is a "[...] power of resisting by which each every body, [...], preserves in its state either of resting or moving uniformly straight forward" subjected to one restriction. (NEWTON, 1999, p. 404). This said restriction can be interpreted from "as much as in it lies", meaning that there might be some circumstances in which the body is prevented from remaining in its state.

It seems reasonable to assume that Newton was not quite lose from the Aristotelian view that every motion requires a cause, or a "motor" which the Paris Terminists believed to reside within the body. This group of thinkers from Paris which include, among others, Jean Buridan, Albert of Saxony and Nicole Oresme began to study and investigate the causes of motion during the development of the new natural philosophy. One of the main contributions of the Paris Terminists is the so called the *impetus*, which emerged from the works of Jean Buridan.

Sometimes the term "terminism" equates with nominalism, a metaphysical view according to which general or abstract terms and predicates exist, while universal or abstract objects which are sometimes thought to correspond to these terms do not. It is primarily a position on the problem of universals, which dates back at least to Plato, and is opposed to realism. Although the Paris Terminists were indeed nominalists in their logic, in order to thoroughly analyse motion, their perspective towards the natural philosophy ultimately rejected the ideas of nominalism and ended up furnishing a very realistic view. The obvious relevance of the Paris Terminists in Natural Philosophy explains why their writings were largely spread through Europe and brought up to discussions regarding Physics. Their influence is such that some historians refer to them as "precursors parisians of Galileo".

As well as Galileo, Newton's work was highly influenced by the writings and concepts previously developed, what might be the reason he could not fully abandoned the Aristotelian ideas when describing a "force" of inertia. However, it is essential for us to comprehend that a radical rejection of the old in order to have a reconstruction from the core is nearly if not completely impossible in science. That impossibility is due to the fact that the individual trying to rebuild any idea, theory or concept finds himself struggling against what he grew up on. In other words, a first attempt to explain the new is generally elaborated and phrased in the old terms, for mankind preserves its culture, traditions and is embedded in the symbols and concepts of a specific period. It is, then, absolutely natural that the development of science occurs in a non linear fashion, requiring the successors of a thinker to make the ideas more and more clear with the successive exposition and studies.

Many philosophers and historians agree that Newton has also read and been influenced by Galileo's *Dialogo sopra i due massimi sistemi del mondo*. In a dialogue between Simplicio and Salvatio, Galileo states that in a perfectly smooth horizontal surface in today's terms: frictionless a perfectly round sphere would not be neither accelerated nor decelerated, which means that the motion would be as perpetual as the surface would let it be. That was the first time a "Law of Inertia" although Galileo never mentions this word was enunciated. (GALILEI, 1997).

Acccording to Júlio César Ribeiro Vasconcelos (2005), one of the phrases that may be referred to as the expression of Galileo's principle of inertia is: "[...] it is reasonable to expect that whatever degree of speed is found in a piece of furniture, it is by its nature indelibly printed provided that the external causes of acceleration or retardation are removed ". (GALILEI,1997). In general, Galileo stated that: "A moving body on a flat surface will continue in the same direction and at constant speed unless disturbed." Although speed today is usually referred to as a vector, in Galileo's time it was still a scalar, and therefore he believed that a uniform circular motion was natural, and that a body in such a movement would continue to have it until a force disturbed him. This argument served to explain the motion of the Earth around the Sun, since the force of gravity only came later with Newton. It was also with Newton that the speed came to be written as a vector, and the principle came to have the current interpretation.

(GALILEI *apud* VASCONCELOS, 2005, p. 399-400).

The term *vis insita*, as already mentioned, was not coined by Newton, but rather was an expression largely in use at his time. It appears in many books and articles of the period, such as Immortality of the soul a volume by Henry More, who is known to have been an important author to the growth of Newton's thoughts. In this book the reference is found in "*innatam quandam vim vel qualitatem corporibus terrestribus insitam*", literally meaning "innate force implanted in earthly bodies", although the term "*insitus*", translated as "implanted", had also an usage dating back to Cicero where it describes what is "inherent". In Kepler's writings we can also find said term frequently. Another occurrence of this expression is found in the handbook of Aristotelian philosophy by Magirus, which was studied by Newton in Cambridge. (DIJKSTERHUIS, 1961).

On the other hand, the forthcoming of the term "inertia" in Newton's life is not as clear as his possible source of learning about *vis insita*. Cohen (1970) claims he found a clue to Newton's encounter with this term. In "*De Gavitatione*", in which Newton essentially critiques Descartes writings, there are references to an exchange of letters between Descartes and Mersenne. He found out that one of the subjects of these letters was "inertia", which means that this is a source of knowledge where this term is seen in a physical context.

Although this might have been the source of Newton's acquaintance with the idea of inertia, he did not followed Descartes' trail. To the latter, it meant something that would tend to make a body come to rest when the "impressed" force ceased to act, which means that matter could not continue to move once there was no force acting upon it. Whilst Newton stated that his concept of "*vis inertia*" was the property that bodies have of preserving their state, either staying at rest or in uniform rectilinear motion. It is noteworthy that the contemporary concept of inertia in classical mechanics is exactly the one Newton introduced, albeit we have dropped the vis nomenclature.

When Descartes wrote about inertia in his letters, he presumably did as an answer to Mersenne's questions about Kepler's use of the term, however his name is never mentioned. That is, probably, the reason why Newton did not know who introduced the concept into the study of motion.

Leibniz delivered harsh criticism towards Newton, accusing him of stealing Kepler's term. Kepler, however, stood by Descartes' explanation: the body tendency to rest after the force no longer is acting on it.

Differences between "body" in the XVII and in the XXI centuries

When reading the *Principia*, we come across the word "body" several times, either in the definitions or in the Laws. However, as has happened to the term "density", in none of the published books Newton gave an explicit definition to the concept of "body", not even a clue to what might be a body's possible nature. In this particular case, we should consider the time in which he wrote it and the most common usage of terms before trying to criticize his, sometimes assumed, lack of skills in the axiomatization field. (MACH, 1960).

If the XXI century reader wishes to avoid misinterpretations when reading the *Principia*, it is crucial to take the nous of the period thinkers into account. In other words, we can not, under any circumstance, understand the terms that appears in Newton's writings by the definitions known nowadays in classical mechanics.

Newton's descriptions of bodies are restricted most of the time to their dynamical properties. They are basically defined as the objects upon which forces act and that are subjects to the Laws of Motion. By establishing this subjection in the characterization of the term, the nature of Newton's bodies seem to be completely dependent on the restraints given by the Laws of Motion. Which means that these bodies don't need any specific geometrical properties. Today we understand the word "body" essentially as an extended object, which means that it is described as having boundaries, volume and mass. Whenever we want to refer to an infinitely small object for analysis purposes, we use the concept of "point mass". This latter concept was not known by the time Newton was writing the *Principia*, that is the reason why one can not expect modern terms or explanations to appear in the definitions. It was throughout the subsequents studies that the Physics as we know today emerged.

The development of mechanics has forced the replacement of the word "body" by the expression mentioned above: "point mass", or "material point". The necessity of replacing said word is clearly exemplified when we bring up Newton's *vis inertia*, the property that "bodies", have of preserving their state, either staying at rest or in uniform rectilinear motion. In today's terms, the appropriate way to state this concept is to restrict it to a point mass. However, it can be generalized to any particle system and to an extended object in particular by saying that in the absence of external forces the center of mass remains at rest or in a uniform rectilinear motion, i.e., there is no information regarding the motion of the whole body concerning the center of mass.

Although neither the term "center of mass" nor "center of gravity" is the main topic of this section, we find essential to discuss the conceptual difference between them.

Center of gravity is a point on which, either for an extended body, either for a system of point masses not necessarily materially connected or bodies, a single force named total gravity force of the system produces a torque that is identical to the sum of the individual torques of all the gravity forces acting upon the different parts of the system. In other words, if we are interested in describing the rotational dynamics of a given system, we can consider that the total weight is concentrated and acting upon the center of gravity. (SILVEIRA, 2014).

While the first case gives useful information about translational motion, allowing us to ignores the non,vanishing extension of the system, the second aids the description of the system rotation and has to fully acknowledge its spacial distribution.

It is noteworthy that both, although with clear reflections in the objective world, are abstract concepts, i.e., one should never look for a point where, concretely, all the masses are concentrated or a point where, de facto, all the gravity forces are acting. A clear example is the case of a homogeneous doughnut, in which the center of mass and the center of gravity are located in its void region, where there is no dough.

There is no reason, a priori, to state that both CM and CG are coincident points. However, it can be proved that, in a homogeneous gravitational field, they actually coincide. That is certainly the reason so many people use these two different concepts interchangeably as synonyms.

The battle for the conquest of the force

During the 17th century, a period in which many of the greatest thinkers of all time lived, the development of new concepts and theories concerning the natural philosophy was taking place all over the world. However, among the terms and words being coined, reinterpreted and thought of, there was a, not always, silent battle to define the force.

At that time, a major concern among those great minds was to find what would be the fundamental quantity what we name "force" in this section to the complete description of motion. The concept of vis was still hugely influenced by Aristotle. He stated that any object in motion had a vis inherent, therefore to him, force would be the measure of motion itself, that is, the change.

If we talk about a battle between great minds, it is logical to assume that it would not be so easy to find an agreement between what had been said before by Descartes, what Leibniz believed and what Newton would state in the *Principia*. That being said, we must understand what each of these thinkers stood up for. In general terms, Descartes' force was based on shock, contact, impact; Leibniz's force was the pure kinetic energy. In both cases the force was something internal to the body. Only Newton was able to establish that force was something that acted. The newtonian force can be summed up in something that one body exerts on another, by contact or not.

Religion played a very important role in the early development of science, mainly because almost all of the theories concerning nature were thought by men that were inserted in a society that strongly believed in God. Descartes sees God as the first cause of motion a conserved quantity in the universe, and force as being nothing but the quantity of that motion. To Descartes, force was the product of the mass of an object and its velocity, not vector.

Leibniz would disprove Descartes and his followers. Whilst for Descartes nature was matter in motion, for Leibniz it was energy in motion, that is the reason why he believed that the study of the force should be that of the dynamics. In order to measure the force, Leibniz thought of experiments of falling and rising bodies collisions with a surface, and concluded that the force would not be the measure of the impact. (PONCZEK, 2000).

We should first take into account what had been previously discovered by other philosophers, such as Galileo. Leibniz already knew that the final velocity of a falling body did not depended on its weight, but on the square root of the initial height. Leibniz argument is simply the following statement: Take two bodies, A and B. Let the mass of A be four times smaller than that of B. A is standing at a place four times higher than B. At the moment they collide with the ground, the force should be exactly the same. Descartes can not be right about the force being the quantity of motion – mv – because than, since it is known that the final velocity is the square root of the initial height, the force of A would not be equal the force of B. In order to have the same force, the ratio between the velocities should be four and not two as it would have been by Descartes' definition. That is the moment when Leibniz proves that the true measure of the force is the product of mass and velocity square, and he calls it *vis viva*. (PONCZEK, 2000).

The basis of Leibniz argument is the causal equivalence between bodies A and B

– or whichever two bodies – standing in an initial height that is inversely proportional to their masses. That can be inferred through the study of static, which, historically, is a point in his favor, since this field was already well established at the time. (SILVEIRA, 2014).

The battle was, then, between Leibniz's *vis viva*, the quantity mv^2, and Descartes' quantity of motion, mv. Both had as many defenders as they had attackers. Their theories were fighting each other in order to obtain the status of real measure of motion. It is essential to have in mind that people were not only influenced by previous thinkers, as Aristotle, but by the doctrines that prevailed at the time. It was only when Newton got into this debate that some ideas started to grow. The concept of force not being inherent, but an external act instead was not created out of the blue. The development of science is given through many controversy, discussions, debates, ideas and theories.

Before going into Newton's definition of force, we shall understand what was the theory in which the Natural Philosophy was then based on. The metaphysical and methodological framework of the time was the mechanical philosophy. As we have already mentioned before, the mechanical philosophy was basically the theory that every action upon a body should happen through contact. Newton thinks of a force that is external to the body, and that could also act upon distance.

Newton's concept of force is, then, philosophically different from the ones of Leibniz and Descartes. Despite the conceptual difference, to Newton and to Leibniz the force was the only cause to the change in the motion of the body. This theory is based on the fact that the force is neither originated in the body nor extinguished in it. Moreover, the conceptions respectively of Descartes and Leibniz were connected with conservation laws.

In definition IV Newton defines the impressed force as being the:

> [...] action exerted on the body to change its state either of resting or of moving uniformly straight line. This force consists solely in the action and does not remain in a body after the action has ceased. For a body preserveres in any new state solely by the force inertia. Moreover, there are various sources os impressed force, such as percussion, pressure, or centripetal force. (NEWTON, 1999, p. 405).

Whenever he says something about a center, for example when defining the centripetal force as being the one that compels bodies towards a center, he is only thinking about a mathematical point.

There are two known interpretations regarding the position Newton embraces. The first one is that Newton is trying to quantify the study of motion, in order to allow phenomena to be correctly evaluated even though he is not determining their occurrence. The second interpretation states that this determination can not be allowed in the scope of Natural Philosophy, thus every mention to force must be understood in an instrumental way. (CHIBENI, 2013).

As we can see, there has been many ideas and concepts of what would be force and there is no right or wrong when it comes to theories in science. There are just the ones that fit better in a certain period, taking into account the knowledge available. According to the development of methods, studies and comprehension, the concepts are being molded until the ones we use today. In the following section we will be able to discuss what is the force in the modern studies of mechanics. (COHEN, 1970).

Are the modern books accurate about Newton's Law?

Before we discuss the modern books, we should learn what is written in the *Principia*. In Book I, Newton begins stating the definitions, some of which we have already mentioned before, then there is the Scholium on space, time and motion. In this scholium, Newton exposes his view on these concepts, but he does not define them, since he believes they are well known. What he actually does is distinguish them, in relative and absolute, mathematical and common, true and apparent. He does that based on his assumption that there is a prejudice raised from the way "common people" conceive said terms.

We shall not analyse these concepts in this article, once they are material for a whole other work. It is, however, important to mention Newton's belief in the absolute space, as we can see in the *Principia*: Absolute space, of its own nature without reference to anything external, always remains always homogeneous and immovable. Relative space is any movable mesure or dimension of this absolute space; [...]. (NEWTON, 1999, p. 408-409).

After this discussion, Newton states his Laws of Motion. There are three laws, and then the Corolary. We will discuss Law I and Law II in this section.

Newton's First Law according to *Principia*

We find in the *Principia* the first Law written as: "Every body perseveres in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed" (NEWTON, 1999, p. 416), with the following commentary: "Projectiles persevere in its motions, except insofar as they are retarded by the resistance of the air and are impelled downwards by the force of gravity." (NEWTON, 1999, p. 416).

As it has already been mentioned before, the first thing one should do in order to understand the *Principia* is to get away of the moorings keeping us in the modern conceptions of Physics. In the analyses of this Law should be no different. If we try to reallocate this statement ipsis literis into contemporary studies we may argue about the lack of information, or even about its general vagueness.

What is this information that we might find lacking? As a modern reader, it is natural to question: In what frame of reference is this true? There are two possible methods to settle this matter.

The first way to solve the question is to postulate the existence of an Alpha-Body, a motionless and imperceptible body standing somewhere – or basically everywhere, that would be the frame of reference in which a body perseveres in its state in the absence of external forces.

Another approach is the one that gives another interpretation to what Newton wrote. The new formulation then, postulates the existence of a inertial frame of reference. Said frame is the one in which free particles – particles that are under no influence of external forces perseveres in their state, either being of rest or of uniform motion in a right line. With the establishment of one inertial frame, there are, automatically, infinite other inertial frames. This direct occurrence happens only because any other frame of reference that is in a uniform rectilinear translational motion or at rest in relation to the inertial frame is inertial as well.

That was not a issue to Newton worry about because as he had previously stated in the *Principia*, he was a believer of absolute space and time.

Axiom II and Newton's Second Law

As we have done in the previous subsection, let's take a look in what is written in the *Principia* about Law II: “A chage in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.” (NEWTON, 1999, p. 416).

And then there’s the comment following the Law:

> If some force generates any motion, twice the force will generate twice the motion, and three times the force will generate three times the motion, whether the force is impressed all at once or successively by degrees. And if the body was previously moving, the new motion (since motion is aways in the same direction as the generative force) is added to the original motion if that motion was in the same direction or is substracted from the original motion if it was in the opposite direction or, if it was in an oblique direction, is combined obliquely and compounded with it according to the directions of both motions. (NEWTON, 1999, p. 416-417).

We shall begin with the second part of the Law II, since this is the non controversial portion of this Axiom. It is simply stated that the change of motion or quantity of motion, as he describes in Definition II occurs in the same direction as the impressed force. He reinforces this idea in the comment below the Law where he also determines that a new momentum is compounded with the one that already exists to result in a total momentum. (COHEN, 1970).

The first part of the law is the one which we need to look at in a careful way. If one wants to interpret the first information contained in the Law as a constant force producing a constant acceleration and their magnitudes proportionality, one needs to read “alteration” or change for that matters as a “rate of change”. Thinking of it as a rate of change and not just “change”, *mutatio*, as he wrote, we have:

$$F = dp/dt = d/dt\,(m\,v) \qquad (1)$$

Being m constant, we actually find:

$$F = m.aF = m.a \qquad (2)$$

What is exactly what one would expect to find in the *Principia*.

However, why should we have to understand the Law in a way that it is not written? In other words, why should we see “change” as a “rate of change”? That would be perfectly acceptable if the statement as it is was impossible to make sense of. Is that the case?

Before we begin, it must be clear that the reader needs to have an open mind to enter this discussion. One should not already depart from the idea that Law II gives us $F = m.a$ and disregard or contempt the arguments that may lead to any other final conclusion.

Although we have mentioned Newton's issues in the axiomatization field, it is highly unlikely that he would miss a word that changes completely the sense of the axiom. With that being said, we shall now proceed to the study of the law exactly how it is stated.

Once it is forsaken the view that the word "force" in Law II states about a force that is being continuously acting, through an impartial reading it is possible to see that it appears to be about an impulsive force. An immediate change of momentum accrues from that impulsive force. This argument may cause the reader to think of what in modern terms would be written as:

$$I = \Delta (m\, v) \qquad (3)$$

If we take the approach of the impulsive forces, we see that Newton never wrote in the *Principia* the relation between a constant force generating a constant acceleration. One may find the reason for that lying in the fact that Newton probably assumed his readers would know that if a constant force is acting upon a particle, its velocity would increase by equal amounts in equal times.

Our interpretations issues

When revising some Physics books, either the ones used in high school or those directed to university students, sometimes we see some problems when it comes to the interpretation of Newton's Laws.

We strongly believe that this is the reason why history and philosophy of science should be studied not only by students of said areas but also by teachers and students of all science fields. In order to comprehend the development of concepts, terms and Laws we deal with everyday. And to avoid misinterpretations that seems to be so common nowadays.

The first major problem we found when looking through books was the fact that the majority of them just take for granted what it is the frame of reference. It is basically assumed that it is known, when in fact, it is not. Law I of *Principia* is commonly written in modern books without explaining what a inertial frame of reference is, and that is exactly what the first Law postulates: the existence of these inertial frames.

When writing that Law I is nothing but how a body reacts after every force has ceased acting, one is neglecting the main point: in which frame of reference does that happen? And if answering that happens in a inertial frame, one gets into another question that can not be answered: what is an inertial frame? When interpreting the first law as the establishment of the existence of inertial frames of reference, and those being the ones in which free particles perseveres in its state of rest or uniform rectilinear motion one no longer have problems with the questions mentioned above.

Another problem we face is the argument that Law I is a particular case of Law II. Again, that is a problem of interpretation. If one understands the first Law not as the establishment of the existence of inertial frames, Law I can be seen as: no force applied, no change in the momentum, therefore the uniform rectilinear motion. However, as we have already said before, Law I is about postulating the existence of inertial frames!

One of the arguments that corroborates the fact that Law I could never be a particular case of Law II is that the first states that a force being exerted is sufficient for a change in the momentum and it is a necessary condition as well. While Law II only gives the force as a sufficient condition for this change and quantifies it. Also, it is essential to have in mind that, in a modern understanding, Law II is only valid in a inertial reference frame. As we have already seen, it is Law I that postulates the existence of such frames. Therefore, it makes no sense to argue about an allegedly futility of Law I, once Law II is only valid in a frame which existence is guaranteed by Law I alone.

It can not be stated a priori only through the second law that whenever there is acceleration a force is, necessarily, acting upon the body, what is a direct consequence of this law is that when a force is exerted, there will be an acceleration. It is Law I that gives us what we need to conclude that the presence of acceleration means there is a force being employed.

Still concerning Law II, many people expect to find the modern form, $F = m.a$ explicitly written in the *Principia*. However, as we have before, this is nowhere in the book. Out of curiosity, this equation appears for the first time in Leonhard Euler's writings in 1750.

Final Coonsiderations

Reintering the objectives of this article, we think that our the main purpose is to provide, as mentioned before, physics teachers, students or enthusiasts with tools and arguments to comprehend the concepts and terms used beyond the common misinterpretations so widely spread was satisfactorily achieved.

After discussing some of the controversial points in the *Principia*, we feel secure to state that one of the most important steps in order to understand Newton's, or any other writing from other periods, writings is to read them keeping in mind in which period that was written. It is not logical to apply conclusions, premises or ideas from today into what has been written centuries before.

There is no doubt Newton has changed the path of natural philosophy with so many great contributions to the field, but it is almost impossible for the modern reader to study classic mechanics through the *Principia*. The problem not only relies in the axiomatization or the lack of it, but in the words used that differs a lot from the ones we use and know nowadays, the concepts and mathematical terms that has been developed through the years and the knowledge available at the time.

We were able to bring to thinking the fact that the question about the circularity of Definition I could be solved if one takes an approach based on historic and philosophical facts instead of only looking at it with the concepts already established today. We could also discuss about the terms that were used and created back then and how with the development of the Natural Philosophy they have been changing. And that one should not expect to find the modern meaning in the words when reading the *Principia.*

Mach apud Cohen (1999, p. 89) severely criticized the way in which Newton defined the concept of mass. The first criticism of this important philosopher, whose ideas inspired the famous Vienna Circle, was related to the fact that Newton had not previously defined the concept of density appropriately. According to Mach apud Cohen (1999, p. 89), if the density was defined as the mass divided by the volume unit, d = m / V, how could the mass be defined as being proportional to density and volume simultaneously? In Mach's words: "Let us first point out, with respect to the concept of mass, that Newton's formulation, defining mass as the quantity of matter of a body, determined by the product of volume and density, is unhappy. Since we can only define density as the mass of unit volume, the circle is obvious. (MARCH apud COHEN, 1999, p. 89-90). However, it does not seem obvious to us that Newton did not perceive the circularity of his definition of mass. On Newton's "supposed" deception, commentator Henry Crew, told us that:

> [...] in Newton's time, density and specific gravity were used as synonyms, and the density of water was arbitrarily taken as unitary. The three fundamental units employed (...) were, therefore, density, length and time, instead of ours, mass, length, and time. In such a system, it is both natural and logically permissible to define mass in terms of density. (CREW apud COHEN, 1999, p. 95).

Therefore, contrary to what Mach apud Cohen (1999, p. 89-90) thought, perhaps Newton had not been wrong about the relation of the quantity of matter to the density and volume, for in the seventeenth century the density was defined in a completely different way than it would be in the 20th century. In another passage of the Mathematical Principles of Natural Philosophy, in "Corollary 4" of "Proposition 6," "Theorem 6," Newton mentioned and in a sense defined density in the following statement: If all solid particles of all bodies are of the same density, and can not be rarefied without pores, then a void or vacuum must be admitted. By bodies of the same density I mean those whose inertia is in the proportion of their volumes. (NEWTON, 1999, p. 810). We note that Newton did not assert that the "small solid particles" were all the same size, but stated that they had the same density. If he had said that they were of the same size, it would imply that their densities would be proportional to the number-counting-of these "small solid particles" in equal volumes, confirming the circularity of the proposition as suggested by Mach apud Cohen (1999, p. 89- 90).

When bringing the dispute for the conquest of the force, we hope to have been able to open the minds of the readers to see how the development of the science is permeated by discussions, arguments, opposed ideas and it is only through a process of thinking, arguing, questioning and interpreting that some idea is established.

The comparison between modern books and what was actually written in the *Principia* is of great value as one can realize that it is not found in Newton’s writings exactly what it is taught nowadays. And it should not be. We shall reinforce that the knowledge goes through many modifications to get to what we know today, but it is extremely important to understand how to interpret such a significant work. That shall be our goal as teachers or researches: supply everybody that is influenced by our work with what is necessary to learn how to interpret and to have a critical sense about what is read, or, in other words, it is fundamental to understand one should never board on a idea without giving it much thought.

One should never overlook the great importance of such work. It is of a high historic relevance, but has had also an amazing influence to the development of Physics.

That is the main reason this subject should never be unappreciated. In order to avoid so many misinterpretations concerning such significant physics concepts such as the first and second, teachers should learn about the history of what they teach as well as learn how to read the *Principia* and confronting it critically.

We presented and discussed some points of Newton's writings, but it is essential to highlight that this is not a closed discussion with nothing left to discover or to give a new interpretation. Science is always finding a way to grow, to change and to become wider.

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